OPTIMIZATION TECHNIQUES FOR THE QUADRATIC ASSIGNMENT PROBLEMS

A Synopsis

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1. INTRODUCTION

Assigning tasks or jobs to workers of variant sort of abilities is one of the toughest work in different fields of knowledge. Very often under proper assignments of jobs to workers results in much reduced cost or increased profit according to the nature of the problems. The utmost priority is to get maximum profit (or minimum cost) out of minimum input given. This problem is considered as classical assignment problem. There should be one to one mapping between the jobs and workers available for these jobs. The assignment problem is one of the first studied combinatorial optimization problems. Although the name "Assignment problem" is first appeared in a paper of Votaw and Orden [56] in 1952, but the beginning of the development of practical solution methods for and variations on the classical assignment problem recognized after the publication of "The Hungarian Method for the Assignment Problem" by Kuhn [36] in 1955.

Consider a cost matrix associated with n workers and n jobs in table 1

		Jobs				
		J_1	.....	J_j	.....	J_n
Workers	W_1	C_{11}	.....	C_{1j}	.....	C_{1n}
	⋮	⋮	⋮	⋮	⋮	⋮
	W_i	C_{i1}	.....	C_{ij}	.....	C_{in}
	⋮	⋮	⋮	⋮	⋮	⋮
	W_n	C_{n1}	.....	C_{nj}	.....	C_{nn}

Table 1

Where

C_{ij} = cost of assigning j^{th} job to i^{th} worker.

$$X_{ij} = \begin{cases} 1 & \text{If } j^{\text{th}} \text{ job is assigned to } i^{th} \text{ worker} \\ 0 & \text{Otherwise} \end{cases}$$

The problem is to assign jobs to workers (one-one correspondence) so as to minimize the total cost of performing all the jobs. The mathematical formulation of the classical assignment problem is

$$\text{Min Z} = \sum_{i=1}^{n}\sum_{j=1}^{n} C_{ij} X_{ij}$$

Subject to the constraints

$$\sum_{i=1}^{n} X_{ij} = 1 \qquad \forall\, j = 1,2, \dots \dots \dots . n$$

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$X_{ij} \in \{0\,, 1\}$

Quadratic assignment problems (QAP) are easy to describe, not so easy to formulate, and very hard to solve [46]. It can be seen through an example of the planning of landing and taking off flights at the terminals of the airport. We see daily the landing and taking off flights at the airport. Suppose there are n flights and n terminals. Passengers get down from one flight at one terminal and moving away to get another flight at another terminal. Such type of movement of passengers can easily be observed at any international airport. Sometimes chaotic situation occurs at the airport by the passengers. Suppose there are n flights $F_1,\ F_2, \dots, F_i, \dots, F_j, \dots, F_n$ are to land at n terminals $D_1, D_2, \dots, D_k, \dots, D_l, \dots, D_n$ each at one. Let i^{th} flight F_i is to land at k^{th} terminal D_k and j^{th} flight F_j is to land at l^{th} terminal D_l. Let f_{ij} be the number of passengers moving from i^{th} flight to j^{th} flight and d_{kl} be the distance from k^{th} terminal to l^{th} terminal. Then total travelled distance by moving f_{ij} number of passengers from D_k to D_l is $f_{ij}d_{kl}$ also total travelled distance by moving f_{ji} number of passengers from D_l to D_k is $f_{ji}d_{lk}$. Then total two way travelled distance between D_k and D_l terminals is $f_{ij}d_{kl} + f_{ji}d_{lk}$ .The problem is to assign the flights to the terminals (one-one correspondence) so as to minimize total travelled distance by the passengers. In general moving passengers termed as flow, terminals as locations and flights as facilities.

We may depict QAP by considering a pair of facilities (i, j) assigned at a pair of locations (k, l) as shown in figure 1.

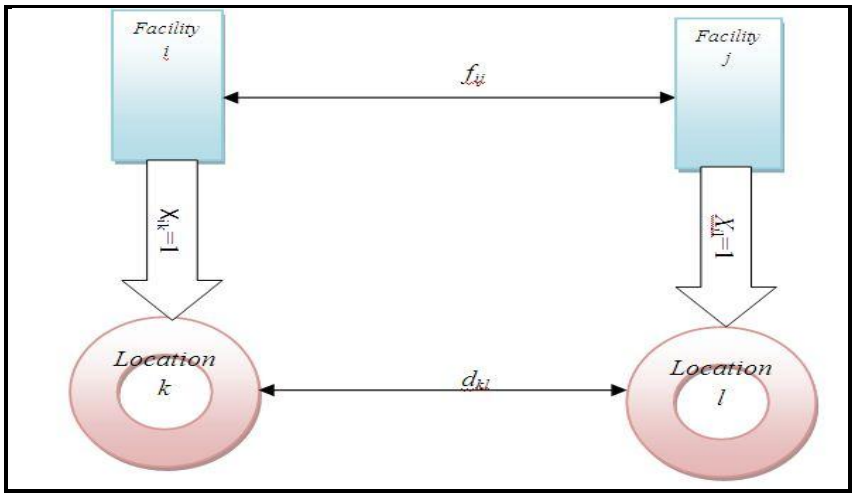

Figure 1: Exhibiting the Facility Layout

Note that the flow from i^{th} facility to j^{th} facility may or may not be equal to that of j^{th} facility to i^{th} facility as per the arrangements of the landing and taking off flights and the need of the passengers.. In flight-terminal example the number of persons moving from i^{th} to j^{th} flight may or may not be equal to that of j^{th} to i^{th} flight, resulting the flow matrix may or may not be symmetric. Distance from k^{th} to l^{th} location may or may not be equal to that of l^{th} to k^{th} location because it has also be seen that sometimes two way paths of different lengths are assigned between the two locations for movement results into the distance matrix which may or may not be symmetric.

The QAP (Quadratic Assignment Problem) was first introduced by Koopmans and Beckmann [35] in 1957 as model for assigning a set of economic activities to a set of locations provided no single activity is assigned to two different locations meaning that there should be one to one mapping between the locations and activities.

The QAP is a discrete optimization problem which can be found in many areas of studies such as Economics, Operations Research and Engineering. It is one of the most difficult combinatorial optimization problems. Several NP-hard problems can be modeled as a QAP.

The applications of Quadratic Assignment Problem can be found in backboard wiring [52], scheduling problems [25], typewriter keyboards and control panel [45], hospital planning [22],

archeology [34], economics problem [31], statistical analysis [32], forest parks [12], analysis of reaction chemistry [23], numerical analysis [13], placement of electronic components [42, 20] etc.

2. MATHEMATICAL FORMULATION OF QAP

To formulate problem in mathematical form we consider a flow, distance and assignment matrix as follows:

	F_1	.....	F_j	.....	F_n
F_1	f_{11}	.....	f_{1j}	.....	f_{1n}
⋮	⋮	⋮	⋮	⋮	⋮
F_i	f_{i1}	.....	f_{ij}	.....	f_{in}
⋮	⋮	⋮	⋮	⋮	⋮
F_n	f_{n1}	.....	f_{nj}	.....	f_{nn}

Table 2: The Flow Matrix

	D_1	.....	D_l	.....	D_n
D_1	d_{11}	.....	d_{1l}	.....	d_{1n}
⋮	⋮	⋮	⋮	⋮	⋮
D_k	d_{k1}	.....	d_{kl}	.....	d_{kn}
⋮	⋮	⋮	⋮	⋮	⋮
D_n	d_{n1}	.....	d_{nl}	.....	d_{nn}

Table 3: The Distance Matrix

		Locations						
		L_1	.....	L_k	.....	L_l	....	L_n
	F_1	X_{11}	.....	X_{1k}	.....	X_{1l}	.....	X_{1n}
	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
	F_i	X_{i1}	.....	X_{ik}	.....	X_{il}	.....	X_{in}
Facilities	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
	F_j	X_{j1}	.....	X_{jk}	.....	X_{jl}	.....	X_{jn}
	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
	F_n	X_{n1}	.....	X_{nk}	.....	X_{nl}	....	X_{nn}

Table 4: The Assignment Matrix

If the flow and distance matrices are given by table 2 and table 3 then their cost matrix for 3 facilities and 3 locations in particular can be formed as follows:

$$\left[\begin{array}{ccc|ccc|ccc}
f_{11}d_{11} & * & * & * & f_{11}d_{22} & * & * & * & f_{11}d_{33} \\
* & f_{12}d_{12} & f_{12}d_{13} & f_{12}d_{21} & * & f_{12}d_{23} & f_{12}d_{31} & f_{12}d_{32} & * \\
* & f_{13}d_{12} & f_{13}d_{13} & f_{13}d_{21} & * & f_{13}d_{23} & f_{13}d_{31} & f_{13}d_{32} & * \\
\hline
* & f_{21}d_{12} & f_{21}d_{13} & f_{21}d_{21} & * & f_{21}d_{23} & f_{21}d_{31} & f_{21}d_{32} & * \\
f_{22}d_{11} & * & * & * & f_{22}d_{22} & * & * & * & f_{22}d_{33} \\
* & f_{23}d_{12} & f_{23}d_{13} & f_{23}d_{21} & * & f_{23}d_{23} & f_{23}d_{31} & f_{23}d_{32} & * \\
\hline
* & f_{31}d_{12} & f_{31}d_{13} & f_{31}d_{21} & * & f_{31}d_{23} & f_{31}d_{31} & f_{31}d_{32} & * \\
* & f_{32}d_{12} & f_{32}d_{13} & f_{32}d_{21} & * & f_{32}d_{23} & f_{32}d_{31} & f_{32}d_{32} & * \\
f_{33}d_{11} & * & * & * & f_{33}d_{22} & * & * & * & f_{33}d_{33}
\end{array}\right]$$

Table 5: Cost Matrix for $n = 3$

Note that there are some asterisks in the cost matrix and these asterisks show disallowed elements. For example, consider $f_{21}d_{22}$. It means it is the travelled distance between the facility 2 and facility 1 given that facility 2 is assigned to location 2 and facility 1 assigned to location 2. Since there is one-one correspondence between facility and location then this travelled distance is meaningless. In given cost matrix there are n^2 sub matrices and each sub matrix contains an element of the form $f_{ij}d_{kl}, i = j, k = l$ termed as 'leader' element and elements of the form $f_{ij}d_{kl}$, $f_{ji}d_{lk}$ are together called 'complementary' cost elements because if $f_{ij}d_{kl}$ involve in assignment cost then $f_{ji}d_{lk}$ also involve in assignment cost. Let us see some terminology involved in QAP.

f_{ij} = Flow from facility *i* to facility *j*

$F = [f_{ij}]$ = The flow matrix

d_{kl} = Distance from location *k* to location *l*

$D = [d_{kl}]$ = The distance matrix.

n = Number of facilities = Number of locations.

The mathematical formulation of problem is to find X_{ik} and X_{jl} which minimizes the objective function satisfying the constraints *i.e.*

$$Min\ Z = \sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{k=1}^{n}\sum_{l=1}^{n} f_{ij}\, d_{kl} X_{ik} X_{jl}$$

$$\sum_{k=1}^{n} X_{ik} = 1 \qquad \forall\, i = 1, 2, \ldots\ldots n$$

$$\sum_{l=1}^{n} X_{jl} = 1 \qquad \forall\, j = 1, 2, \ldots\ldots n$$

$X_{ik} \in \{0\,,1\}$ and $X_{jl} \in \{0\,,1\}$

$$X_{ik} = \begin{cases} 1 & \text{If } i^{\text{th}} \text{ facility is assigned to } k^{th} \text{ location} \\ 0 & \text{Otherwise} \end{cases}$$

$$X_{jl} = \begin{cases} 1 & \text{If } j^{\text{th}} \text{ facility is assigned to } l^{th} \text{ location} \\ 0 & \text{Otherwise} \end{cases}$$

3. SURVEY OF PUBLISHED RESEARCH

After reading some published survey [41, 55, 57] on QAP and some works related to QAP and Linear Assignment Problem (LAP) by eminent researchers the works done on QAP so far are as follows:

The Quadratic Assignment Problem was first introduced by T.C. Koopmans and M. Beckmann [35] in 1957 as a mathematical model related to economic activities.

The Gilmore-Lawler bound (GLB) presented by Gilmore (1962) [26] and Lawler (1963) [39] is one of the best known lower bound for QAP. Gilmore focused on the Koopmans-Beckmann formulation while Lawler was concerned with the general form. Lawler presented branch and bound procedure for solving the QAP by converting it into equivalent LAPs. The lower bound is calculated by solving at most $n^2 + 1$ LAPs for each node. Evidently the approach is impractical for all but the smallest problem.

The history of solving QAP exactly centers now on the famous Nugent *et. al.* Problems. In 1968, Nugent, Vollmann and Ruml [44] proposed a set of problems of size 5,6,7,8,12,15,20 and 30, and noted for their difficulty.

Sahni and Gonzales (1976) [49] had shown that it is NP- hard and that, unless P=NP, it is not possible to find an f-approximation algorithm for constant f. In 1978 Burkard and Stratmann [15] optimally solved the Nugent's 12 problem in 29.325 seconds on a CDC cyber 76 machine. They used a branch and bound perturbation algorithm, again utilizing the popular GL Lower bound.

Bazara and Sherali (1980) [10] reformulated the QAP as a 0-1 mixed linear program with n^2 integer variables, $n^2(n-1)^2/2$ continuous variables and $2n^2 + 2n$ constraints. The researchers applied Benders' Decomposition to this formulation but were unable to solve problem larger than $n = 6$ optimally within reasonable time.

Bazara and Sherali (1982) [9] used the formulation of the Quadratic Assignment Problem as that of minimizing a concave quadratic function over the assignment polytope. Cutting plane procedures are proposed for this problem. In particular efficient methods of generating cuts have been discussed. In this method, a lower bound derived on the number of cuts needed for termination indicating that the cutting plane procedures would require a huge computational effort for exact solution. However, several heuristics methods have been derived from the cutting planes produce optimal and good quality solution at early stage in the search process.

Although exact procedures for determining optimal solution of QAP also presented by Land (1963) [38], Gavett and Plyter (1966) [24], Cabot and Francis (1970) [16], Kaufman and Broeckx (1978) [37] Bazara and Elshafei (1979) [11], Edwards (1980) [21] and Kaku and Thompson (1986) [33] but they were unable to solve the problems of size >10 optimally. Adams *et.al.* (2007) [2] proposed a level-2 reformulation and solved problem up to size 30.

The QAP was one of the first problems solved by metaheuristic methods first conceived in the (1980). Burkard and Rendl (1984) [14] proposed a simulated annealing procedure that was able to find much better solutions than all the previously designed heuristic methods. The branch and bound based heuristic also given by Bazara and Kirca (1983) [8] using the Gilmore – Lawler method incorporated with the property referred to as 'mirror image'. They were able to solve a hospital layout planning of size $n = 19$ optimally within 110 seconds on CDC-70.

In 1987 Catharine Roucairol [47] has developed a parallel branch and bound algorithm for the QAP. This algorithm has been implemented on an asynchronous multiprocessor machine with

shared memory for problem of $n \geq 10$. Very good results are obtained for the problem of size $n = 12$.

P. Hahn and T. Grant (1998) [30] described bounding technique for the Quadratic Assignment Problem based upon a dual formulation, they extends the Hungarian method (1957) [43] for linear assignment problem to Quadratic Assignment Problem. The QAP iteratively transformed in a series of equivalent QAPs leading to an increasing sequence of lower bounds for the original problem. The values of the transformed objective function are the corresponding values of the old objective function, shifted by some amount C. If there exists feasible solution such that its value in transformed problem is zero then C is optimal value for the original QAP and current feasible solution is optimal. The transformations are iteratively applied until no significant increase in constant C is found, resulting in the so called Dual Procedure. Earlier some works were also carried out on lower bounding technique by Adams and Johnson (1994) [1], Assad and Xu (1985) [7], Carraresi and Mallucelli (1992) [17], Gilmore (1962) [26], Lawler (1963) [39], Roucairol (1979) [48] and Hahn *et.al.* [27, 28, 29].

Connolly (1990) [18] proposed an improved annealing scheme. His method is too easy to set up, since the user has to select the number of iterations; and all other parameters are automatically computed. During the same time period, Skorin-Kapov (1990) [51] proposed a tabu search, with fewer parameters and running 'n' times faster than the previous implementation. Heuristic GRASP (greedy randomized adaptive search procedure) was proposed by Li, Pardalos and Resende (1994) [40]. Genetic algorithms have also been proposed by Tate and Smith (1995) [54].

Taillard (1991)[53] proposed a more robust tabu search. Even though Taillard's method was proposed over 12 years ago, it remains one of the most efficient for some problems. Ahuja, Orlin and Tiwari (2000) [5] have given more efficient another heuristic. Similar research has been conducted by Ahyaningsih and Sitompul (2015) [6].

A Data-guided Lexisearch Algorithm for the Asymmetric Travelling Salesman Problem has been developed by Ahmed Z .H. (2011) [3]. In this method bias has been removed by subtracting the row minima and after those column minima. The matrix obtained after subtraction is called modified cost matrix. After that Alphabet table based on the modified cost matrix is constructed

and lower bound is calculated. The objective of lower bound is to skip as many sub problems in the search procedure as possible.

Ahmed Z. H. (2014) [4] also developed a Data- guided Lexisearch Algorithm for quadratic assignment problem. In this method it has been shown that before applying lexisearch algorithm minor preprocessing of data improves computational time significantly.

In 2014, A Maximin Zero Suffix Method has been developed by Sharma *et. al.*[50]for solving the linear assignment problem. To obtain the minimum assignment cost, least cost cell being searched in each row and each column. In the assignment table, after subtracting the lowest cost of each row from all the costs of corresponding row and then subtracting the lowest cost of each column from all the costs of corresponding column at least one zero in each row and each column are obtained. The lowest cost cells in each row and in each column are detected by these zeros. On minimum cost distribution, zero cells (Lowest cost cells) should be assigned as far as possible. Assignments are made to the cells having maximum suffix value of zero. The suffix value of a zero is the maximum of the lowest cost in its row and the lowest cost in its column excluding itself.

The practical applications of QAP are found in [12,13,20,22,23,25,31,32,34,42,45,52].

In spite of such a combinatorial optimization problem recognized earlier and some researches going on, not a proper technique in the form of algorithm has been developed to solve QAP till now.

4. PROPOSED WORK

After reading a number of research papers and articles on QAP and related problems, we got inspiration to project some research directions. In these directions we propose to develop some techniques in the form of algorithm to solve QAP. The directions (methodology of research) are as follows:

(i) The cost matrix comprises n^2 sub matrices. It has been found that each sub matrix of cost matrix and the cost matrix itself exhibits same assignment pattern. Using this fact, assignment is needed in only one sub matrix which explores the complete assignment. The sub matrix having the greatest suffix value is to be assigned for getting solution nearer to the optimal one as suggested by Sharma *et. al.* [50].

(ii) Travelling salesman problem can be viewed as a particular case of the QAP, when the flow between any pair of facility is one. Since bias removal method is applied in [3] is useful for solving travelling salesman problem, reformulation of the QAP based on bias removal method is supposed to be effective for developing the technique for solving QAP.

(iii) An algorithm will be developed using cutting plane method for solving QAP. For this, firstly Integer programming formulation for QAP will be discussed and then technique for solving QAP to be developed. The cutting plane method will start with a feasible solution. In order to find optimal solution a cut will be generated which remove the current solution but no other solution which has a quadratic objective value better than the current best. After applying the cut the procedure is then to search another point which is feasible to the cut generated so far. If no such point exists then current best solution will be declared optimal.

(iv) Applying lower bounds as suggested by Peter Hahn and Thomas Grant [30] to extend the Hungarian method for the linear assignment problem, a technique will be developed for solving QAP. The major drawback of this method is that it gives unequal preference to upper and lower index sub matrices for given complementary cost pair, yielding subsequent imbalances in the leader lower bounds.

(v) An algorithm for solving QAP may be developed by branching and bounding the problem *i.e.* branch and bound procedure. The lower bound may easily calculated by matching largest value of $[f_{ij}]$ to the minimum value of $[d_{kl}]$ then block the cell which is already matched and search next largest of $[f_{ij}]$ to match next minimum of $[d_{kl}]$ until all the elements are matched. It is hoped that cell blocking procedure may be useful to find lower bound to QAP. After calculating lower bound we will further try to get optimal solution.

Some other directions may be searched to develop techniques for solving the QAP, if needed.

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